A Bi-Level Optimization Formulation of Priority Service Pricing

Yuting Mou, Anthony Papavasiliou, *Member, IEEE,* and Philippe Chevalier

***Abstract*—Priority service pricing is a promising approach for mobilizing residential demand response, by offering electricity as a service with various levels of reliability. Higher levels of reliability correspond to higher prices. Proper pricing guarantees that consumers self-select a level of reliability that corresponds to the reliability that the system can offer. However, the traditional theory for menu design is based on numerous stringent assumptions, which may not be respected in practice, such as well-behaved convex cost functions for generators. In this paper, we design a priority service menu as the equilibrium solution to a Stackelberg game, which is modeled as a bi-level optimization problem involving a vertically integrated utility and consumers. We reformulate the equilibrium as a mixed-integer problem. As a consequence of this approach, we can integrate the menu design problem within a day-ahead unit commitment model. This allows us to design a menu which exactly meets the profit requirements of a firm. The approach is illustrated on a toy numerical example as well as a large-scale model of the Belgian power market.**

***Index Terms*—demand response, priority service pricing, bi-level optimization, ADMM, dual decomposition**

NOMENCLATURE

A. Sets

$\mathcal{L}, L$	Set of consumers and its cardinality
$\mathcal{T}, T$	Set of time periods and its cardinality
$\mathcal{I}, I$	Set of options and its cardinality
Ω	Set of scenarios
$\mathcal{G}$	Set of generators

B. Parameters

Θ_t	Dynamic profile of consumption
V_l	Valuation of consumer l [€/MWh]
$\bar{D}_l$	Average demand of consumer l [MW]
VB_i	Valuation breakpoint, $i = 0 \dots I$ [€/MWh]
V_i	Average valuation of group i [€/MWh]
D_i	Demand of group i [MW]
K	Slop of demand function
$\Pi_\star$	Profit target [€]
Π^+	Upper bound on prices in the menu [€/MWh]
P_ω	Probability of scenario ω
$S_{t,\omega}$	Solar production at hour t in scenario ω [MW]
$W_{t,\omega}$	Wind production at hour t in scenario ω [MW]
$\bar{\mu}_{l,i}$	Inferred consumer subscription decision

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C. Variables

π_i	Price of option i [€/MWh]
r_i	Reliability of option i [%]
$\mu_{l,i}$	Binary decision of consumer l for option i
$y_{l,i}$	Auxiliary variable to represent $\pi_i \cdot \mu_{l,i}$
$w_{l,i}$	Auxiliary variable to represent $r_i \cdot \mu_{l,i}$
$s_{l,i}$	The subscription quantity of consumer l under option i [MW]
γ_l	Lagrange multiplier
s_i	The total subscription quantity under option i [MW]
$d_{i,t,\omega}$	Supply to option i at hour t in scenario ω [MW]
$c_{t,\omega}$	Total costs at hour t in scenario ω [€]
$p_{g,t,\omega}$	Production of generator g at hour t in scenario ω [MW]
$m_{g,t,\omega}$	Start up decision of generator g at hour t in scenario ω
$n_{g,t,\omega}$	Shut down decision of generator g at hour t in scenario ω
$o_{g,t,\omega}$	Unit commitment decision of generator g at hour t in scenario ω
$\mathbf{m}$	Compact form of $m_{g,t,\omega}$, similarly for $\mathbf{n}$, $\mathbf{o}$, $\mathbf{p}$, $\mathbf{c}$, $\mathbf{d}$, $\mathbf{y}$, $\boldsymbol{\gamma}$, $\mathbf{r}$, $\boldsymbol{\pi}$, $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$
$\mathbf{s}$	Compact form of $s_{l,i}$

D. Functions

$h_{t,\omega}$	Cost function including production costs, startup and minimum load costs
$f_{g,\omega}$	Constraints of unit commitment problems, including minimum up and down times, ramp rates and production limits
SW	Social welfare function
CS	Consumer surplus function

I. INTRODUCTION

The mobilization of demand-side resources is an essential requirement for enabling the large-scale integration of renewable resources. These resources can be instrumental in mitigating numerous system and market operation challenges resulting from the integration of renewable resources. On a short-term operational basis, demand-side flexibility can serve towards balancing the system on an instantaneous basis [1], and can contribute towards resolving numerous operational challenges related to the integration of renewable resources (e.g. ramping, the negative correlation of renewable supply with demand, and the wear and tear of thermal units due to startups). In the long term, price-responsive consumers can contribute towards signaling scarcity in capacity and flexibility

[2]. This is an essential step towards tackling the missing money problem [3] by properly remunerating conventional resources that offer valuable services to the system.

A. Demand Response Potential

There exist numerous economic studies in the literature which demonstrate the value of mobilizing demand-side flexibility. Studies by Faruqui et al. [4], [5] demonstrate the benefits of dynamic pricing in terms of preventing peak capacity investments. Recent studies by [6] support these observations by quantifying the economic value of flexibility (including the flexibility of distributed loads) at approximately 8 billion British pounds per year for the UK alone. These savings result from reduced short-term fuel costs, lower long-term investment costs in power generation capacity, and the deferral of avoidable reinforcements of the transmission and distribution grid.

A recent study by Gils [7] has demonstrated that the residential sector is a significant source of demand-side flexibility. Serendipitously, some of the most energy-intensive appliances in households are also among the most flexible, in the sense that deferring or interrupting them may have minor impacts on the perceived quality of service. Such appliances include dish washers, laundry machines, dryers, and in the future possibly electric vehicles.

Although residential demand response presents very promising opportunities, it has failed to deliver its promise in electricity markets [2]. The development of adequate business models for engaging flexible consumers [8] is an essential step in successfully enlisting demand-side flexibility in system operations. In the two extremes of the wide spectrum of options for mobilizing demand response [9], one identifies price-based methods and quantity-based methods. Real-time pricing, the golden standard of price-based methods, suffers from the fact that it places an excessive information overhead on residential consumers, who lack the attention span and economic incentives to voluntarily engage in the process of procuring electricity in real time. On the other hand, quantity-based methods such as direct load control are perceived as being excessively intrusive.

B. Priority Service Pricing

The approach which is considered in this paper is the quality differentiation of electricity service, which can be viewed as a compromise between price and quantity-based methods that attempts to combine the best of both worlds. Quality-differentiated service traces its theoretical origins in non-linear pricing [10], and is inspired by success stories in the telecommunications and information technology sectors. The promise of this approach as a viable paradigm for massively scalable demand response is exemplified by the notable amount of research that has been conducted recently in variations of the basic concept [11]–[16].

The idea of quality differentiation is to treat electricity as a service, with different levels of quality, as opposed to a commodity that is procured in a time-varying real-time price. Higher quality implies a higher price, in an analogous way to how monthly Internet subscriptions with a larger bandwidth demand a higher price.

Priority service pricing [17], which is the most basic form of quality differentiated pricing and the focus of this paper, has been applied in different industries [18]. "*For example, in transportation systems, railways offer express and regular freight services. The postal system offers priority and regular mail services. Most service industries provide some form of priority service in order to reduce waiting times for customers with high waiting costs. Other examples include computer service bureaus, job shops, and express toll roads.*" In power service, an early example of priority service pricing was the Pacific Gas and Electric tariff for large industrial customers that included explicit options for curtailable and interruptible power service [10].

In this study, we focus on the application of interruptible power service to residential consumers, since the residential sector arguably places the greatest premium on *simple* service offerings that do not require excessive attention overhead. The idea is to offer residential consumers a menu of price–reliability pairs for *strips* of *power*.

Households decide how to allocate devices to strips. One way to implement this allocation is by tagging plugs with colors that correspond to reliability levels in the home [2], either manually or automatically through a home energy router. Thus, households enroll to an electricity service with an intuitive interpretation, while preserving control on their household consumption. The necessary control and communication technology for implementing priority service pricing requires a means of tagging plugs according to reliability levels, an ability to monitor slices of different reliability in real time (e.g. 5-to-15 minute intervals), and an energy router that can receive control signals from a utility and relay them to plugs with the appropriate reliability tags [2], or undertake the color tagging on its own.

By selecting plans, consumers reveal their valuation for power. These valuations can be aggregated and bid into the wholesale electricity market. This promotes price discovery and an efficient allocation of resources under tight system conditions, which are expected to occur increasingly frequently in the future due to the integration of renewable resources.

C. Contribution

An important challenge of priority service is the pricing of different menu options. In order to appreciate the challenge, consider two extremes. In one extreme, an aggregator prices all levels of reliability at a very low price. In this case, all consumers enroll to the option with the highest level of reliability. This is undesirable, since the aggregator would neither be able to deliver the promised level of reliability, nor to discriminate consumers according to their valuation. On the other extreme, if the aggregator prices all levels of reliability at an excessively high price, then no consumers enroll voluntarily. The theory for designing a menu optimally has been developed by [17] and relies on strong assumptions. The appeal of the theory is that it only requires *aggregate* statistical information about the population, which is becoming increasingly available through real-world demand response pilots [5], [19]–[21].

There has been further development of priority service pricing based on the seminal paper of Chao [17], including menus differentiated by duration and reliability, and menus that account for capacity expansion [22]. Chao [23] extends the consumer model to a two-stage model, consisting of a subscription stage and an actual consumption decision stage if service is curtailed. Campaign [24] considers a profit-maximizing aggregator in a competitive setting. However, the aforementioned models involve stringent assumptions, are more difficult to implement in practice, and rely on closed-form solutions that do not exploit the capabilities of powerful commercial optimization solvers.

In this paper we revisit Chao's theory and extend it to a more realistic set of assumptions. In doing so, we couple the menu design problem with production simulation models based on unit commitment. Embedding the menu design problem with unit commitment allows us to override a number of weaknesses of the traditional theory, and creates numerous advantages from an analysis standpoint. *(i)* We are able to clarify exactly what the service of a certain reliability level means[1]. *(ii)* We are able to introduce profit targets that the menu should achieve seamlessly in our model, which are essential for cost-benefit analyses of smart meter deployment [5]. *(iii)* Coupling realistic models of demand response with unit commitment, which has been attempted in past literature under less realistic settings such as real-time pricing or fixed retail pricing [25]–[27], is essential for capturing the operational benefits of demand response (mitigation of ramping constraints, reduction of non-convex costs related to startup and min load, etc.). *(iv)* The temporal coupling in the production model (due to min up/down time constraints, and ramp rates) is captured and correspondingly, the consumers are modeled as being dynamic explicitly. *(v)* The framework that we develop allows us to revisit various generalizations of quality differentiated service, including menus that are differentiated by duration and reliability [28] and menus that account for capacity expansion [22].

II. Modeling the Menu Design Problem as a Stackelberg Equilibrium

In what follows, we will cast the menu design problem as a Stackelberg game. The leader in the Stackelberg game is the producer who designs the menu. The followers are the consumers, who react to the menu offered by the leader. The information asymmetry arises from the fact that the followers have private knowledge of their *type*, whereas the leader is limited to statistical information about the distribution of types in the population (e.g. from market surveys). The interesting aspect of the model is that the leader integrates the optimal reaction of the followers to the menu design problem. This gives rise to a mathematical program with equilibrium constraints. Although such problems are generally challenging, we exploit the specific structure of the game in order to cast the problem as a mixed integer linear program. This allows us to incorporate realistic production constraints to the problem, which are absent from the traditional literature on priority service pricing. We thus arrive to a model that integrates menu design with unit commitment.

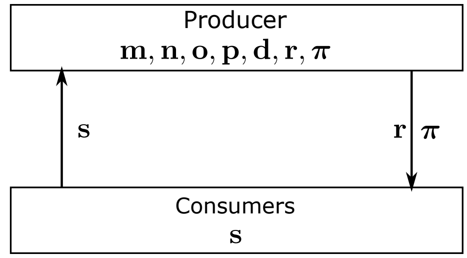

Fig. 1. Interaction between the producer and consumers in the bi-level model.

A high-level description of the bi-level model is provided in Eqs. (1)-(5). The model is further illustrated in Fig. 1.

$$\max_{\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p},\mathbf{d},\mathbf{r},\boldsymbol{\pi}} \quad SW(\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p},\mathbf{d}) \tag{1}$$

$$\text{subject to: } (\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p},\mathbf{d}) \in \mathcal{X} \tag{2}$$

$$\mathbf{r} = \phi(\mathbf{d},\mathbf{s}^\star) \tag{3}$$

$$\Pi_\star = \psi(\mathbf{m},\mathbf{o},\mathbf{p},\mathbf{s}^\star,\boldsymbol{\pi}) \tag{4}$$

$$\mathbf{s}^\star \in \arg\max_{\mathbf{s}} \{CS(\mathbf{r},\boldsymbol{\pi}) : \mathbf{s} \in \mathcal{S}\} \tag{5}$$

The variables $\mathbf{m}$, $\mathbf{n}$, $\mathbf{o}$, $\mathbf{p}$ correspond to startup and shutdown decisions, unit commitment and power generation. The subscription quantity of each consumer to each option is indicated by $\mathbf{s}$, while the supply to each option is indicated by $\mathbf{d}$. The reliability and price of the menu options is denoted by $\mathbf{r}$ and $\boldsymbol{\pi}$, respectively. The profit target of the producer[2] is denoted by $\Pi_\star$.

The function SW in Eq. (1) is the objective of the producer, which is to maximize social welfare. Eq. (2) defines the technical constraints of the producer. Constraint (3) expresses the fact that the price menu which is designed by the producer needs to deliver a promised level of reliability $\mathbf{r}$, which is affected by how consumers react to the offered menu through their subscription decision $\mathbf{s}^\star$. Condition (4) further requires that the menu be designed in such a way that the profit target $\Pi_\star$ is reached. Consumers decide on their subscription by maximizing their surplus CS, as indicated by Eq. (5).

In what follows, we introduce the lower-level consumer model and the upper-level producer model in detail. We then present the bi-level model and its reformulation as an MILP.

A. The Consumer Model

The starting point of nonlinear pricing is to capture information asymmetry by assigning a type to consumers [10]. This is private information, in the sense that the producer cannot know a priori the type of a given consumer (although this

[1]Five minutes of interruption every hour and one month of *straight* interruption every year imply large differences for consumer comfort, even if both are characterized by a reliability of 11/12.

[2]Nonlinear pricing theory typically considers the setting of a regulated monopoly that needs to recover certain amount of costs, such as investment costs, by imposing a target on the profit, see chapter 5 of [10] for details.

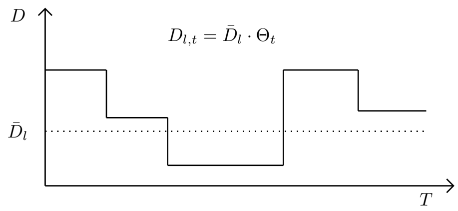

Fig. 2. A consumer of type l follows a load profile $D_{l,t}$. Since loads are assumed to be synchronized, this profile can be expressed as $D_{l,t} = \bar{D}_l \cdot \Theta_t$, where Θ_t is the dynamic profile of the system and $\bar{D}_l$ is the average electricity consumption of type l.

information is revealed to the producer after the consumers self-select their preferred menu option). In priority service pricing, types correspond to *valuation* for power.

Before presenting the consumer model, we point out two strong assumptions[3] that are typically employed in the priority service pricing literature [28] regarding the demand side of the model. The first assumption is that the priority ranking of consumers for power remains constant over time. The second assumption is that loads are synchronized.

Given the above assumptions, we arrive to the following consumer model. Given a set of consumer types, $\mathcal{L}$, a consumer of type $l \in \mathcal{L}$ is characterized by a valuation V_l, which represents the priority ranking of a consumer, and remains constant over the whole horizon according to the first assumption[4], in order to arrive to a single menu. Without loss of generality, we order consumers as $V_l < V_{l+1}$. In period t, the consumer of type l requires $D_{l,t}$ units of power, where $D_{l,t} = \bar{D}_l \cdot \Theta_t$. Here, $\bar{D}_l$ corresponds to the average load of type l and Θ_t corresponds to the dynamic profile of consumption [5]. This dynamic profile is identical for all consumers and the same as the load profile of the residential sector, due to our second assumption that loads are synchronized. The synchronization of loads can be relaxed, however that would place significant additional information requirements on the producer to know the dynamic profile of each different type. The load profile of the residential sector is a time series indicating the hourly electricity consumption of the residential sector. The concept is depicted in Fig. 2. By definition, $\sum_{t\in\mathcal{T}} \Theta_t = T$, where T is the number of time periods over which we are designing the menu.

As a follower, the consumer selects service options from a menu with a set of options $\mathcal{I}$. Each of the options corresponds to a unit of electricity consumption with reliability r_i and price π_i. Reliability r_i is defined as the fraction of energy offered to option i, divided by the energy requested under option i. More specifically, denote the subscription quantity to option i as s_i and the supply to this option at period t and scenario ω as $d_{i,t,\omega}$. In choosing option i, consumer l essentially procures $s_{l,i} \cdot \Theta_t$ following a *profile* Θ_t, so that the energy that is requested under option i is given as $\sum_{t\in\mathcal{T}} s_i \cdot \Theta_t$. The energy that is actually offered to option i is calculated as $\sum_{\omega\in\Omega} P_\omega \sum_{t\in\mathcal{T}} d_{i,t,\omega}$, where P_ω is the probability of scenario ω. Thus, r_i is expressed as $\sum_{\omega\in\Omega} P_\omega \sum_{t\in\mathcal{T}} d_{i,t,\omega} / \sum_{t\in\mathcal{T}} s_i \cdot \Theta_t$. This is the origin of constraint (23), which we present later.

Since π_i is the hourly price of option i, the total payment of subscribing for a unit of power under option i for the entire horizon T of the contract amounts to $\pi_i \cdot T$.

Concretely, given r_i and π_i from the upper level producer model, the optimization problem of the consumer of type l can be described as follows:

$$\max_{s_{l,i}} \; V_l \cdot \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{I}} r_i \cdot s_{l,i} \cdot \Theta_t - \sum_{i\in\mathcal{I}} s_{l,i} \cdot \pi_i \cdot T \quad (6)$$

$$\text{subject to} : \sum_{i\in\mathcal{I}} s_{l,i} \cdot \Theta_t \leq D_{l,t} \quad (7)$$

$$s_{l,i} \geq 0, i \in \mathcal{I} \quad (8)$$

The variable $s_{l,i}$ indicates the amount of power that consumer l allocates to option i. The first term in the objective function indicates the benefit of a risk-neutral consumer for this profile. The second term in the objective function corresponds to the payment that needs to be submitted to the producer in order to secure this service. Constraint (10) requires that the total subscription of the consumer should not exceed the load of the consumer. Since $\sum_{t\in\mathcal{T}} \Theta_t = T$ and $D_{l,t} = \bar{D}_l \cdot \Theta_t$, we can rewrite the model equivalently as

$$(CP_l) : \max_{s_{l,i}} \; \sum_{i\in\mathcal{I}} (V_l \cdot r_i \cdot s_{l,i} - s_{l,i} \cdot \pi_i) \quad (9)$$

$$(\gamma_l) : \sum_{i\in\mathcal{I}} s_{l,i} \leq \bar{D}_l \quad (10)$$

$$s_{l,i} \geq 0, i \in \mathcal{I} \quad (11)$$

We wish to use the optimality conditions of this problem as constraints of the producer problem. We do this in order to capture the fact that, when designing a menu, the producer accounts for the optimal reaction of the consumers. In complementarity form, these conditions will be problematic. Since CP_l is an LP, we wish to express the optimality conditions of the consumer problem as a collection of primal feasibility, dual feasibility, and strong duality conditions [29], [30], and we further exploit the special structure of the consumer problem so as to describe the optimal subscription of the consumer as a binary variable. As we illustrate later in the paper, this is essential for our MILP formulation of the bilevel problem.

[3] These assumptions are *approximations*, and they entail a certain loss of information, which is the price to pay for the simplicity of the offered menu. If these assumptions were true, then priority service pricing could be reproduced equivalently by having agents submit their valuation for power once and for all to the aggregator (see section III of [17] for a discussion on the equivalence between priority service pricing and spot pricing).

[4] Note that this is an *approximate* model of consumers that is employed by the producer in order to design the menu. The true behavior of consumers corresponds to valuations that evolve dynamically, and possibly in a non-synchronized fashion. Such valuations could be elicited by real-time pricing auctions, however the entire motivation of this work is that the complexity of real-time pricing as a means of eliciting the true valuation information places a prohibitive informational overhead on residential consumers, and priority service pricing corresponds to a second-best alternative.

[5] The introduction of Θ_t is inspired by [28] . This enables us to generalize the consumer model to a dynamic setting explicitly, so that the temporal coupling on the production side can be captured.

The dual of (CP_l) can be expressed as:

$$(CD_l) : \min_{\gamma_l} \ \gamma_l \cdot \bar{D}_l \tag{12}$$

$$\gamma_l \geq r_i \cdot V_l - \pi_i, i \in \mathcal{I} \tag{13}$$

$$\gamma_l \geq 0 \tag{14}$$

The interpretation of γ_l is that it captures the surplus that the consumer achieves by selecting the best option. Strong duality requires that

$$\gamma_l \cdot \bar{D}_l = \sum_{i \in \mathcal{I}} r_i \cdot s_{l,i} \cdot V_l - \sum_{i \in \mathcal{I}} \pi_i \cdot s_{l,i}. \tag{15}$$

Eq. (15) involves bilinear terms when it is treated as a constraint of the reformulated single-level problem, which cannot be dealt with by MILP solvers. We override this problem by showing that the optimal decision of the consumer is binary.

Proposition 1: There exists $\tilde{\mathbf{s}}_l = (\tilde{s}_{l,i}, i \in \mathcal{I})$ with $\tilde{s}_{l,i} \in \{0, \bar{D}_l\}$ which attains the optimal objective function value.

Proof: The KKT conditions of (CP_l) are given by

$$0 \leq s_{l,i} \perp -r_i \cdot V_l + \pi_i + \gamma_l \geq 0 \tag{16}$$

$$0 \leq \gamma_l \perp \bar{D}_l - \sum_i s_{l,i} \geq 0 \tag{17}$$

There are two cases to be considered:

Case 1: If $\bar{D}_l - \sum_i s^\star_{l,i} > 0$, then $\gamma_l = 0$, which implies that consumer l derives zero benefits at the optimal solution, so $\tilde{s}_{l,i} = 0$ for all $i \in \mathcal{I}$ is optimal.

Case 2: If $\bar{D}_l - \sum_{i \in \mathcal{I}} s^\star_{l,i} = 0$, then it suffices to show that if two options are 'active' (in the sense that $s > 0$) then they have an equal payoff, and can therefore be equivalently replaced by a single option. Applying this argument for all options that are active gives the desired conclusion: consider any two options i and j for which $s^\star_{l,i} > 0$ and $s^\star_{l,j} > 0$. Then $-r_i \cdot V_l + \pi_i + \gamma_l = 0$ and $-r_j \cdot V_l + \pi_j + \gamma_l = 0$, and substituting out γ_l, we have $r_i \cdot V_l - \pi_i = r_j \cdot V_l - \pi_j$. ∎

The above proposition implies that $s_{l,i}$ can be expressed as $s_{l,i} = \bar{D}_l \cdot \mu_{l,i}$, where $\mu_{l,i} \in \{0, 1\}$ are binary variables. Thus, Eq. (15) is rewritten as

$$\gamma_l = \sum_{i \in \mathcal{I}} r_i \cdot \mu_{l,i} \cdot V_l - \sum_{i \in \mathcal{I}} \pi_i \cdot \mu_{l,i}. \tag{18}$$

Combined with McCormick envelopes, this reformulation will allow us to cast the lower-level optimality conditions of the Stackelberg game as a set of mixed integer linear constraints. This will be detailed in section II-C.

B. The Producer Model

We follow the standard literature on priority service pricing [10], [17], [28] in assuming a vertical setup where the producer who is responsible for aggregating demand response also owns the production assets of the system. An interesting extension of the present work is to use our mathematical programming reformulation in order to analyze aggregator competition and decentralization of the production decisions. This extension is out of the scope of the present paper.

As a leader of the Stackelberg game, the producer seeks to price reliability so that residential consumers self-select reliability levels which are consistent with the generation mix of the system. Information asymmetry implies that the producer does not know, at the menu design stage, the type (i.e. the valuation) of an individual consumer. Instead, the producer has access to the *distribution* of types in the population. This is exactly the demand function of the system. For the derivation of constraint (25), we will specifically assume an affine demand function of the form $D(v) = -K \cdot v + b$. Note, however, that the priority service model is not limited to affine demand functions. We use a discrete approximation of the demand function, with the valuation breakpoints[6] VB_i $(i = 0 \dots I)$, with the valuation of the first breakpoint corresponding to $VB_0 = 0$ €/MWh. These breakpoints separate consumers into I groups and $(VB_{i-1} + VB_i)/2$ corresponds to the average valuation of consumer group $i \in \mathcal{I}$, while $K \cdot (VB_i - VB_{i-1})$ corresponds to the load (in MW) of group $i \in \mathcal{I}$.*Given* a choice of options by individual consumers, $s^\star_{l,i}$, the producer problem then becomes:

$$(PP) : \max_{d_{i,t,\omega}, \mathbf{m}, \mathbf{n}, \mathbf{o}, \mathbf{p}, r_i, \pi_i} \sum_{\omega \in \Omega} P_\omega \sum_{t \in \mathcal{T}} \Big(- h_{t,\omega}(\mathbf{m}, \mathbf{o}, \mathbf{p}) + \sum_{i \in \mathcal{I}} 0.5 \cdot (VB_{i-1} + VB_i) \cdot d_{i,t,\omega} \Big) \tag{19}$$

subject to:

$$f_{g,\omega}(\mathbf{m}, \mathbf{n}, \mathbf{o}, \mathbf{p}) \leq 0, g \in \mathcal{G}, \omega \in \Omega \tag{20}$$

$$\sum_{i \in \mathcal{I}} d_{i,t,\omega} = \sum_{g \in \mathcal{G}} p_{g,t,\omega} + S_{t,\omega} + W_{t,\omega}, t \in \mathcal{T}, \omega \in \Omega \tag{21}$$

$$d_{i,t,\omega} \leq s_i \cdot \Theta_t, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega \tag{22}$$

$$T \cdot r_i \cdot s_i = \sum_{\omega \in \Omega} P_\omega \sum_{t \in \mathcal{T}} d_{i,t,\omega}, i \in \mathcal{I} \tag{23}$$

$$T \cdot \sum_{i \in \mathcal{I}} s_i \cdot \pi_i - \sum_{\omega \in \Omega} P_\omega \sum_{t \in \mathcal{T}} h_{t,\omega}(\mathbf{m}, \mathbf{o}, \mathbf{p}) = \Pi_\star \tag{24}$$

$$s_i = K \cdot (VB_i - VB_{i-1}), i \in \mathcal{I} \tag{25}$$

$$\sum_{l \in \mathcal{L}} s^\star_{l,i} = s_i, i \in \mathcal{I} \tag{26}$$

$$d_{i,t,\omega}, p_{g,t,\omega} \geq 0, i \in \mathcal{I}, g \in \mathcal{G}, t \in \mathcal{T}, \omega \in \Omega \tag{27}$$

$$m_{g,t,\omega}, n_{g,t,\omega}, o_{g,t,\omega} \in \{0, 1\}, g \in \mathcal{G}, t \in \mathcal{T}, \omega \in \Omega \tag{28}$$

The goal of the producer is to maximize welfare by using the available production assets of the system. The cost is expressed by the function $h_{t,\omega}$, and can include production costs as well as non-convex costs related to startup and minimum load. The variables $\mathbf{m}$, $\mathbf{n}$, $\mathbf{o}$, $\mathbf{p}$ correspond to startup and shutdown decisions, unit commitment and power generation. The set of generators is denoted as $\mathcal{G}$. The generator constraints are expressed by the function $f_{g,\omega}$, and can include standard constraints of unit commitment problems, such as minimum up and down times, ramp rates, startup profiles, production limits, and so on. The quantity of consumers signed up under option i is indicated by s_i. Their hourly supply is $d_{i,t,\omega}$. Constraint (21)

[6] In determining these breakpoints, we follow the standard priority service literature by assuming that these values are determined exogenously. An interesting extension of our model would be to treat VB_i as decision variables, however the producer model presented later would then involve bilinear terms. We leave this extension as an area of future research.

describes demand and supply balance. Note that uncertainty in the model corresponds to a set of solar and wind production scenarios, with the corresponding output of these resources denoted by $S_{t,\omega}$ and $W_{t,\omega}$. Constraint (22) requires that the supply to consumers be limited by their subscription decisions. Constraint (23) determines the reliability of option i as the fraction of energy offered to option i, divided by the energy requested under option i. The producer seeks to achieve a profit target $\Pi_\star$, as indicated in constraint (24). Constraint (25) implies that the subscription quantity of each option is equal to the estimated demand of the corresponding demand group[7]. The subscription quantity of each option is equal to the sum of the subscription quantity of consumers in this option, as indicated by (26). One single menu is designed for the whole horizon, but the model can be easily adapted to offer monthly/seasonal menus.

C. The Bi-Level Model

Given a choice $s^\star_{l,i}$ of menu options by consumer types, the producer model is a welfare maximizing commitment and dispatch of the system, of the sort encountered in the standard unit commitment literature. The delicate task of the producer is to offer a price menu (r_i, π_i) so that consumers' reaction $s^\star_{l,i}$ is compatible with the estimated grouping of consumers indicated by VB_i, while achieving its profit target. We thus revisit the mathematical programs of section II-A and section II-B in order to develop the full bi-level formulation. Concretely, we are interested in expressing the following bilevel problem in MILP form:

$$(Bilevel): \min_{d_{i,t,\omega},\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p},r_i,\pi_i} \sum_{\omega\in\Omega} P_\omega \sum_{t\in\mathcal{T}} \Big(h_{t,\omega}(\mathbf{m},\mathbf{o},\mathbf{p}) - \sum_{i\in\mathcal{I}} 0.5\cdot(VB_{i-1}+VB_i)\cdot d_{i,t,\omega}\Big) \quad (29)$$

$$\text{subject to:}(20)-(28) \quad (30)$$

$$s^\star_{l,i} \in \arg\max_{s_{l,i}}\{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}(V_l\cdot r_i\cdot s_{l,i}\cdot\Theta_t - s_{l,i}\cdot\pi_i): \sum_{i\in\mathcal{I}} s_{l,i}\le \bar{D}_l, s_{l,i}\ge 0, i\in\mathcal{I}\} \quad (31)$$

We reduce the bilevel problem to a single level by appending the equilibrium constraints of the Stackelberg followers to the leader problem. We do so by treating $s_{l,i}$, r_i and π_i as *variables*, and describing the behavior of $s_{l,i}$ as a *function* of (π_i, r_i) through the primal feasibility, dual feasibility, and strong duality conditions of section II-A.

The primal feasibility constraints (10) and (11) and the dual feasibility constraints (13) and (14) can be inserted directly to the bi-level formulation. Instead, the strong duality constraint (18) becomes a bilinear non-convex constraint when r_i and π_i are treated as decision variables.

[7]This constraint makes sure the average valuation of consumers in option i is $0.5\cdot(VB_{i-1}+VB_i)$, which is consistent with (19). Note that we implicitly require that the designed menu induces *all* consumers to select a specific option in the menu. An interesting extension of the present model which we leave for future research is the possibility of designing the menu such that it intentionally induces certain consumers to not select *any* option from the menu.

In order to overcome this challenge, we express constraint (18) equivalently by its McCormick envelope. We do so by noting that the reliability variable is naturally bounded in the interval $0\le r_i\le 1$, and by imposing a price limit on the menu offering, $0\le\pi_i\le\Pi^+$. This allows us to express $\pi_i\cdot\mu_{l,i}$ by a new variable $y_{l,i}$, and $r_i\cdot\mu_{l,i}$ by a new variable $w_{l,i}$. The strong duality constraint (18) for every type $l\in\mathcal{L}$ can then be rewritten as follows:

$$\begin{aligned}
&\gamma_l = \sum_{i\in\mathcal{I}} w_{l,i}\cdot V_l - \sum_{i\in\mathcal{I}} y_{l,i}\\
&y_{l,i}\le\Pi^+\cdot\mu_{l,i},\ y_{l,i}\ge 0,\ y_{l,i}\le\pi_i\\
&y_{l,i}\ge\Pi^+\cdot\mu_{l,i}+\pi_i-\Pi^+\\
&w_{l,i}\le\mu_{l,i},\ w_{l,i}\ge 0,\ w_{l,i}\le r_i,\\
&w_{l,i}\ge\mu_{l,i}+r_i-1.
\end{aligned}$$

We thus arrive to a reformulation of the equilibrium conditions of the lower level as a mixed integer linear set. However, this reformulation results in a significant increase of binary variables and constraints, which results in prohibitive run times for realistic-scale problems. We overcome this challenge by *(i)* using the structure of the lower-level problem in order to propose a set of valid cuts, and *(ii)* noting that the variables $\mu_{l,i}$ are then implied by the constraints of the problem. We first recall theorem 1 of [17]:

Proposition 2: Consider two consumers with valuation V_m and V_n respectively, and denote the optimal choice of reliability and the corresponding price as $r^\star(V_m)=\sum_{i\in\mathcal{I}} r_i\cdot\mu^\star_{m,i}, \pi^\star(V_m)=\sum_{i\in\mathcal{I}}\pi_i\cdot\mu^\star_{m,i}$. If $V_m>V_n$, then we have $r^\star(V_m)\ge r^\star(V_n)$ and $\pi^\star(V_m)\ge\pi^\star(V_n)$. In other words, consumers with higher valuation select more reliable plans and pay more.

Proposition 2 yields the following set of valid cuts:

$$\sum_{i=k}^{I}\mu_{l,i}\le\sum_{i=k}^{I}\mu_{l+1,i},\ l=1..L-1,\ k\in\mathcal{I}$$

These conditions can be understood as follows. Given a consumer of type l and a consumer of type $l+1$ recall from section II-A that we order consumers by increasing valuation, i.e. $V_l<V_{l+1}$), type $l+1$ subscribes to an option which is at least of the same quality as that of l, because of the higher valuation of type $l+1$. This implies that the value of 1 appears in the sequence $\{\mu_{l+1,i}, i\in\mathcal{I}\}$ no later than it does for the sequence $\{\mu_{l,i}, i\in\mathcal{I}\}$, counting from I.

The second observation which allows us to arrive to a computationally tractable model is the observation that a unique solution of the variables $\mu_{l,i}$ is gained from the following set of constraints and the proof is available from [31].

$$\begin{aligned}
&\mu_{l,i}\in\{0,1\}, l\in\mathcal{L}, i\in\mathcal{I}\\
&\sum_{i\in\mathcal{I}}\mu_{l,i}=1, l\in\mathcal{L}\\
&\sum_{l\in\mathcal{L}}\bar{D}_l\cdot\mu_{l,i}=K\cdot(VB_i-VB_{i-1}), i\in\mathcal{I}\\
&\sum_{i=k}^{I}\mu_{l,i}\le\sum_{i=k}^{I}\mu_{l+1,i},\quad l=1..L-1,\ k=1..I-1
\end{aligned}$$

where the first constraint has been established by proposition 1, the second constraint is implied by condition (25) and the fact that the lowest valuation breakpoint is $VB_0 = 0$ €/MWh, the third constraint is condition (25), and the fourth constraint is derived from proposition 2. Note that this observation implies that the variables $\mu_{l,i}$ of the bi-level model can be replaced by fixed values $\bar{\mu}_{l,i}$. Effectively, the collection of these four conditions is an assignment of the highest consumers of highest type to the options of highest reliability. Even though $\mu_{l,i}$ is implied by these conditions, the challenge of designing a price menu that will induce consumers to self-select the corresponding options *remains*. Achieving this consistency is the goal of the reformulated single level problem, which can be written as follows:

$$(MILP): \min_{d_{i,t,\omega},\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p},c_{t,\omega},r_i,\pi_i,w_{l,i},y_{l,i},\gamma_l} \sum_{\omega\in\Omega} P_\omega \sum_{t\in\mathcal{T}} \left(c_{t,\omega} - \sum_{i\in\mathcal{I}} 0.5 \cdot (VB_{i-1} + VB_i) \cdot d_{i,t,\omega} \right) \quad (32)$$

subject to:

$$f_{g,\omega}(\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p}) \leq 0, g \in \mathcal{G}, \omega \in \Omega \quad (33)$$

$$\sum_{i\in\mathcal{I}} d_{i,t,\omega} = \sum_{g\in\mathcal{G}} p_{g,t,\omega} + S_{t,\omega} + W_{t,\omega}, t \in \mathcal{T}, \omega \in \Omega \quad (34)$$

$$d_{i,t,\omega} \leq K \cdot (VB_i - VB_{i-1}) \cdot \Theta_t, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega \quad (35)$$

$$d_{i,t,\omega}, p_{g,t,\omega} \geq 0, i \in \mathcal{I}, g \in \mathcal{G}, t \in \mathcal{T}, \omega \in \Omega \quad (36)$$

$$m_{g,t,\omega}, n_{g,t,\omega}, o_{g,t,\omega} \in \{0,1\}, g \in \mathcal{G}, t \in \mathcal{T}, \omega \in \Omega \quad (37)$$

$$c_{t,\omega} = h_{t,\omega}(\mathbf{m},\mathbf{o},\mathbf{p}), t \in \mathcal{T}, \omega \in \Omega \quad (38)$$

$$(\boldsymbol{\nu}): \; T \cdot \sum_{l\in\mathcal{L}} w_{l,i} \cdot \bar{D}_l - \sum_{\omega\in\Omega} P_\omega \sum_{t\in\mathcal{T}} d_{i,t,\omega} = 0, i \in \mathcal{I} \quad (39)$$

$$(\lambda): \; T \cdot \sum_{i\in\mathcal{I}} \sum_{l\in\mathcal{L}} y_{l,i} \cdot \bar{D}_l - \sum_{\omega\in\Omega} P_\omega \sum_{t\in\mathcal{T}} c_{t,\omega} - \Pi_\star = 0 \quad (40)$$

$$y_{l,i} \leq \Pi^+ \cdot \bar{\mu}_{l,i}, l \in \mathcal{L}, i \in \mathcal{I} \quad (41)$$

$$y_{l,i} \leq \pi_i, l \in \mathcal{L}, i \in \mathcal{I} \quad (42)$$

$$y_{l,i} \geq \Pi^+ \cdot \bar{\mu}_{l,i} + \pi_i - \Pi^+, l \in \mathcal{L}, i \in \mathcal{I} \quad (43)$$

$$w_{l,i} \leq \bar{\mu}_{l,i}, l \in \mathcal{L}, i \in \mathcal{I} \quad (44)$$

$$w_{l,i} \leq r_i, l \in \mathcal{L}, i \in \mathcal{I} \quad (45)$$

$$w_{l,i} \geq \bar{\mu}_{l,i} + r_i - 1, l \in \mathcal{L}, i \in \mathcal{I} \quad (46)$$

$$\gamma_l \geq r_i \cdot V_l - \pi_i, l \in \mathcal{L}, i \in \mathcal{I} \quad (47)$$

$$\gamma_l = \sum_{i\in\mathcal{I}} w_{l,i} \cdot V_l - \sum_{i\in\mathcal{I}} y_{l,i}, l \in \mathcal{L} \quad (48)$$

$$y_{l,i}, w_{l,i}, \gamma_l, r_i, \pi_i \geq 0, l \in \mathcal{L}, i \in \mathcal{I} \quad (49)$$

In this new formulation, s_i has been substituted out. Constraint (35) is the result of substituting constraint (25) in constraint (22). Note that, in constraint (38), we introduce a new set of free variables $c_{t,\omega}$ for representing the cost $h_{t,\omega}(\mathbf{m},\mathbf{o},\mathbf{p})$. Although redundant from a modeling standpoint, these variables will be useful for decomposing the problem, as described in the following section.

III. Decomposition by ADMM

The bilevel model is reformulated as a single-level MILP, which can potentially be solved by commercial solvers. However, the case study of the Belgian market in section IV-B cannot be solved directly, due to the renewable production scenarios. Therefore, in this section we propose a heuristic based on ADMM [32] in order to decompose the problem. The idea of the decomposition is to relax the coupling constraints (39) and (40) so that the unit commitment problem of each scenario can be tackled independently.

A. ADMM Formulation

Concretely, we define $\mathcal{C}_1$ as the set of constraints (33) - (38) that relate to the unit commitment part of the problem and $\mathcal{C}_2$ as the set of constraints (39) - (49) that relate to the consumer problem. We define $\mathbf{x}_1 = (\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p},\mathbf{c},\mathbf{d})$ and $\mathbf{x}_2 = (\mathbf{y},\mathbf{w},\boldsymbol{\gamma},\mathbf{r},\boldsymbol{\pi},\boldsymbol{\mu})$. Our goal in using an ADMM algorithm is to decompose the overall problem to a part that relates to the unit commitment, and to a part that relates to the consumer. The general idea of the approach is to create copies of $d_{i,\omega,t}$, denoted as $\mathbf{d}^x$ and $\mathbf{d}^z$, that are shared between so-called x-updates (the x updates involve unit commitment problems that are decomposable by scenario) and z-updates (the z updates implicate the consumer variables). In creating these clones of the original variables, we can move the complicating constraint (39) to the consumer sub-problem, and decouple the unit commitment sub-problem by scenario. In the same spirit, in order to relax constraint (40), we create copies of the variable $c_{\omega,t}$ that we denote by $\mathbf{c}^x$ and $\mathbf{c}^z$ respectively. These variables are handled by the unit commitment problems (x-updates) and the consumer problems (z-updates) respectively. The general concept is explained in chapter 5 of [32]. An illustration of the algorithm on a toy example is presented in the appendix.

In abstract form, we left-multiply $\mathbf{x}_1$ by a matrix A of appropriate dimension, which gives us $A\mathbf{x}_1 = (\mathbf{c},\mathbf{d})$ and we create a copy $\mathbf{z}$ of $A\mathbf{x}_1$. The problem (MILP) can thus be rewritten in a stylized form as follows :

$$\min_{\mathbf{x}_1,\mathbf{x}_2,\mathbf{z}} \; f(\mathbf{x}_1) \quad (50)$$

$$\text{subject to: } \mathbf{x}_1 \in \mathcal{C}_1 \quad (51)$$

$$(\mathbf{x}_2,\mathbf{z}) \in \mathcal{C}_2 \quad (52)$$

$$A\mathbf{x}_1 - \mathbf{z} = 0 \quad (53)$$

We can define an indicator function g of $\mathcal{C}_2$, and the problem is rewritten as

$$\min_{\mathbf{x}_1\in\mathcal{C}_1,\mathbf{x}_2,\mathbf{z}} \; f(\mathbf{x}_1) + g(\mathbf{x}_2,\mathbf{z}) \quad (54)$$

$$\text{subject to: } A\mathbf{x}_1 - \mathbf{z} = 0 \quad (55)$$

The ADMM iterations can then be expressed as follows:

$$\mathbf{x}_1^{k+1} := \arg\min_{\mathbf{x}_1\in\mathcal{C}_1} \left(f(\mathbf{x}_1) + (\rho/2)\|A\mathbf{x}_1 - \mathbf{z}^k + \mathbf{u}^k\|_2^2 \right) \quad (56)$$

$$(\mathbf{x}_2^{k+1}, \; \mathbf{z}^{k+1}) := \Pi_{\mathcal{C}_2}(A\mathbf{x}_1^{k+1} + \mathbf{u}^k) \quad (57)$$

$$\mathbf{u}^{k+1} := \mathbf{u}^k + A\mathbf{x}_1^{k+1} - \mathbf{z}^{k+1} \quad (58)$$

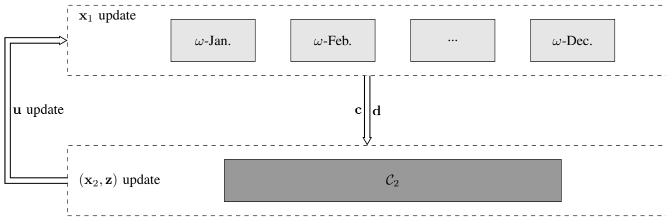

Fig. 3. The application of ADMM as a heuristic for decomposing the reformulated MIP.

where $\Pi_{\mathcal{C}}$ is the projection operator on the set $\mathcal{C}$ and $\mathbf{u}$ are the scaled dual variables. For our specific problem, each element of $A\mathbf{x}_1 - \mathbf{z} = 0$ implicates only variables of a given month and scenario, therefore the regularization term in (56) can be decoupled by month and scenario, which is the original motivation for using a decomposition method.

The scheme is further illustrated in Fig. 3. The light gray blocks correspond to constraints $\mathcal{C}_1$ and the dark gray block corresponds to constraints $\mathcal{C}_2$. Note that the unit commitment problem in each scenario is decomposed into 12 independent monthly unit commitment problems[8], which are solved in parallel, so that there are $12\cdot|\Omega|$ unit commitment subproblems in total. The yellow block updates the $\mathbf{x}_1$ variables. The green block is used for updating $(\mathbf{x}_2, \mathbf{z})$. The variables $\mathbf{u}$ are then used for updating the yellow blocks in the next iteration.

B. Convergence

In practice, the *dual residual* and *primal residual* are used to check the convergence of ADMM for a convex problem. However, the model presented in this paper is non-convex. Therefore, residuals are not a proper indication of convergence in this case, since there is no reason to expect that they should eventually vanish. Instead, we determine the convergence by checking the gap between an upper bound and lower bound of the problem. We derive the upper and lower bound as part of our algorithmic implementation.

1) Upper Bounding by Recovery of Primal Feasible Solutions: The idea of the upper bounding method is to fix the unit commitment part of the problem, and seek a menu design and a set of consumer choices that are consistent with the fixed unit commitment decisions. In doing so, we fix the majority of variables of the original problem, and are left with a relatively light MILP that can potentially yield a feasible solution and an upper bound. Even though there is no convergence guarantee for ADMM when applied to this model, at the end of each ADMM iteration we can fix part of the solution, i.e., $\mathbf{x}_1$, and solve the following problem (PR):

$$\text{(PR)}: \quad (\mathbf{x}_2, \mathbf{z}) \in \mathcal{C}_2 \tag{59}$$

$$A\mathbf{x}_1^\star - \mathbf{z} = 0 \tag{60}$$

If (PR) is feasible, we obtain an upper bound as $f(\mathbf{x}_1^\star)$, otherwise the upper bound returned from the iteration in question is $+\infty$. Note that there is no theoretical guarantee that we can find a feasible solution to (PR). But practically in the case study presented later, we can find a good upper bound after some ADMM iterations. This is because in later iterations, the solution from the unit commitment part of the problem evolves, so that the total cost and price region defined by the consumer choice constraints change, which enable the profit constraint feasible.

[8]Boundary effects are handled by wrapping the monthly commitment problem around itself.

2) Lower Bounding by Dual Decomposition: The following computation is performed once at the outset of the problem, in order to yield a lower bound, before launching the ADMM algorithm. We relax constraints (39) and (40) in problem (MILP) by using the corresponding dual variables $\boldsymbol{\nu}$ and λ, so that the whole problem is decomposed into the following dual subproblems.

Dual subproblem - producer in scenario ω:

$$\min_{d_{i,t,\omega},\mathbf{m},\mathbf{n},\mathbf{o},\mathbf{p}} P_\omega \sum_{t\in\mathcal{T}} \Big(h_{t,\omega}(\mathbf{m},\mathbf{o},\mathbf{p}) - \tag{61}$$

$$\sum_{i\in\mathcal{I}} 0.5 \cdot (VB_{i-1} + VB_i) \cdot d_{i,t,\omega} \Big) \tag{62}$$

$$- \sum_{i\in\mathcal{I}} \nu_i \cdot P_\omega \cdot \sum_{t\in\mathcal{T}} d_{i,t,\omega} \tag{63}$$

$$- \lambda \cdot P_\omega \cdot \sum_{t\in\mathcal{T}} h_{t,\omega}(\mathbf{m},\mathbf{o},\mathbf{p}) \tag{64}$$

$$\text{subject to: } (33) - (37) \tag{65}$$

Dual subproblem - consumer:

$$\min_{r_i,\pi_i,w_{l,i},y_{l,i},\gamma_l} \sum_{i\in\mathcal{I}} \nu_i \cdot T \cdot \sum_{l\in\mathcal{L}} w_{l,i} \cdot \bar{D}_l \tag{66}$$

$$+ \lambda \cdot T \cdot \sum_{i\in\mathcal{I}} \sum_{l\in\mathcal{L}} y_{l,i} \cdot \bar{D}_l - \lambda \cdot \Pi_\star \tag{67}$$

$$\text{subject to: } (41) - (49) \tag{68}$$

This is a standard scenario decomposition of the problem. Many algorithms are available for solving the dual decomposition, such as the sub-gradient method [33], [34], the cutting plane method [35], and bundle methods [36]. In our study, we adopt a standard cutting plane method which is suitable for low-dimensional spaces. Note that, in the implementation of the dual decomposition, the unit commitment problem of each scenario is also decomposed by month. The maximum of the dual function which is obtained from the cutting plane method (which is the sum of the objective values of the dual sub-problems) is a lower bound of (MILP). We can compare this lower bound to the upper bound obtained from the primal feasible solution recovery in order to decide when to terminate the algorithm.

IV. Case Studies

In this section, we present two illustrations of the model. The first one is a toy example used for validation, which compares the closed-form solution provided by priority service pricing theory [17] to the solution of the bi-level model. The second case study is a realistic model of the Belgian power system. For this realistic case study, we compare priority service pricing to real-time pricing and an optimal flat tariff in terms of social welfare, and we analyze the interruption patterns of priority service pricing.

A. A Toy Example

Consider a system with the demand function $D(v) = 1620 - 4v$. The system consists of two generators. The first generator is reliable and has a marginal cost of 65.1 €/MWh, and a capacity of 295 MW. The other generator is unreliable. It is operational with a probability $P_1 = 83.3\%$ and is out of service with a probability $P_2 = 16.7\%$. The second generator has a capacity of 1880 MW and a marginal cost of 0 €/MWh.

Consider the breakpoints $VB_0 = 0$, $VB_1 = 331.25$, $VB_2 = 405$ €/MWh. In this case, 1325 MW subscribe to the first option, and 295 MW subscribe to the second option. The closed-form solution [17] prescribes the following service menu:

$$\pi(r) = \begin{cases} 0 \text{ €/MWh}, & r = 83.3\% \\ 55.3 \text{ €/MWh}, & r = 100\% \end{cases}$$

The profit of the producer amounts to 13106.3 €.

In order to implement the bi-level model in this toy example, we discretize the demand function into 1620 consumer *types*. Given a profit target of 13106.3 €, the model yields a price menu which is identical to that of [17] within one significant digit. If we increase the profit requirement of the firm to 15000 €, we obtain the following price menu from the bi-level model:

$$\pi(r) = \begin{cases} 0.1 \text{ €/MWh}, & r = 81.5\% \\ 61.3 \text{ €/MWh}, & r = 100\% \end{cases}$$

Note that the increased profit target of the producer is largely covered by increasing the price of the second option.

B. The Belgian Power System Model

This section presents a case study of the Belgian system in a forward-looking scenario of the year 2050. A full-year horizon and one-hour resolution is considered.

The conventional generator fleet of the model consists of 55 units. The installed capacity of each technology follows the projected capacity of the year 2050 according to the EU 2050 reference scenario [37]. The technical specifications of the units are available from the website of the Belgian TSO Elia [38]. The installed capacity of conventional generators, which totals 15784 MW, can be broken down as follows: gas (14965 MW), oil (10 MW), biomass (542 MW), and waste (267 MW). The long-term maintenance schedule of units is accounted for by derating the maximum capacity of the units by a certain availability ratio. The availability ratio follows the hourly profiles of 2015 [38].

Wind and solar production profiles corresponding to the years 2013 to 2017 and import profiles for the year 2015 with hourly resolution are collected from [38]. These profiles are scaled up according to the projected value of the year 2050, according to the EU 2050 reference scenario [37]. Ten scenarios of wind and solar production are created, in order to better characterize uncertainty in renewable production[9]. The projected ratio of renewable energy production to total energy production for 2050 is 27.4%, with the peak production amounting to 11690 MW. The projected peak load in 2050 amounts to 18700 MW. It is worth noting that, during some hours, curtailment of renewable production could occur.

The pumped hydro storage in Belgium has a pumping capacity amounting to 1200 MW, while the energy storage capacity of pumped hydro amounts to 5700 MWh. Pumped hydro resources are assumed to have a roundtrip efficiency of 76.5% [39].

The total load profile of year 2015 is also available from [38]. We split this profile into a residential, industrial and commercial load, according to synthetic load profiles [40]. The load profiles are scaled up to the year 2050 according to the EU 2050 reference scenario [37].

Industrial and commercial demand is assumed to be inelastic. Hourly demand functions for residential consumers are assumed to be linear, and are calibrated by assuming an elasticity of -0.5 at the historically observed demand quantity and price for each hour of the data[10].

1) Price Menu Designed According to Chao's Theory: The original theory of priority service pricing relies on a convex cost function, i.e. an economic dispatch model which does not account for startup costs, minimum load costs and minimum capacity constraints of generators. Ignoring these conditions may result in a mismatch between promised and delivered reliability.

In table I we present the results of the menu designed according to the theory of Chao [17]. We have discretized the menu into 5 options. The price and reliability of each option are presented in the first two columns of table I. The fourth and fifth column indicate the average valuation and total demand of each group of types within a given option. Based on this information, a piece-wise constant demand function is used as an input to the *true* unit commitment model of the Belgian system. The *realized* reliability of each option is indicated in the third column of the table. We observe a significant deviation between promised and delivered reliability, especially for the first two options.

2) Performance of the ADMM Algorithm: Fig. 4 presents the convergence of dual decomposition based on the cutting plane method. The lower bound that is obtained by dual decomposition amounts to - 5767.5 million.

ADMM is implemented in Julia and we utilize 120 CPUs on the CÉCI cluster [41] in order to solve the model, with one CPU being dedicated to each subproblem in the x-update of ADMM. Gurobi is chosen as the solver and the MIP gap is set to be 0.1%. The run time for 30 iterations of the algorithm amounts to 2.7 hours. The first feasible solution is

[9]In order to preserve seasonal effects, the scenarios of wind and solar power production are created as follows. In the case of solar, we shuffle the days within the same week. For example, the days in the first week of 2013-2017 are regarded as samples of the same day (35 in total), and then we randomly draw one day from this set. The hourly load factor (production divided by installed capacity) of this day is used in order to derive the production profile of the first day in 2050. For wind, we shuffle the months in the same season.

[10]According to the Tempo program of EDF, the estimated elasticity is between -0.18 to -0.79 [5].

TABLE I
PRICE MENU FOR THE EXAMPLE OF SECTION IV-B BASED ON CHAO'S THEORY.

Price (€/MWh)	Reliability (%)	Realized Reliability (%)	V_i (€/MWh)	D_i (MW)
0.0	21.6	0.6	31.7	436.8
46.0	94.2	93.2	116.9	739.1
52.8	98.2	97.8	231.3	839.9
57.3	99.7	99.7	353.1	839.9
58.3	100.0	100.0	450.5	504.0

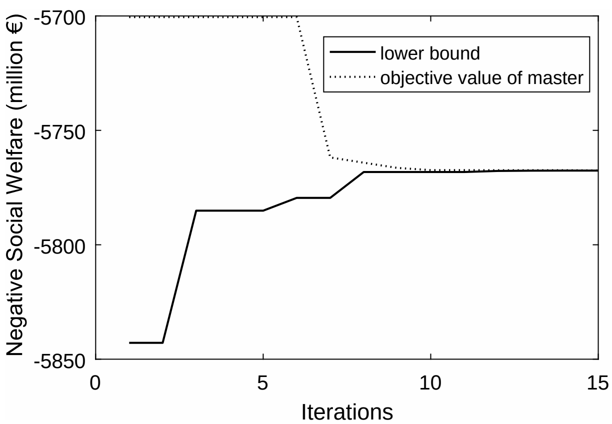

Fig. 4. Convergence of the cutting plane method. The solid curve presents the evolution of the lower bound of the dual function. The dotted curve presents the evolution of the objective value of the master problem in the cutting plane method. A valid cut that limits the objective value of the master problem under -5700.5 million (which is the performance of a flat tariff) is added to the master problem in order to stabilize the first few iterations.

TABLE II
THE PRICE MENU OBTAINED BY ADMM.

Reliability (%)	Realized Reliability (%)	Price (€/MWh)
5.3	5.3	0.0
92.9	92.9	55.6
97.8	97.8	64.0
99.7	99.7	69.6
100.0	100.0	70.6

obtained at the 16th iteration and the corresponding objective value amounts to -5763.4 million. The absolute gap of the algorithm, compared with the lower bound, is 4.1 million, which accounts for 0.34% of the operating costs of serving residential consumers[11]. The required run time for 16 iterations amounts to 1.3 hours. The best solution is achieved at the 27th iteration, with an objective of -5763.5 million. Note that this is very close to the objective function value of the 16th iteration.

3) Welfare Comparison: This section compares the results of priority service pricing with those of the optimal flat tariff and of real-time pricing. Note that the results are based on the assumption of synchronized profiles and the analysis of the case with unsynchronized profiles is available from [31].

Table II presents the promised reliability, realized reliability and price that are obtained from solving the model using our proposed algorithm. It is observed that there is no deviation between the promised reliability and realized reliability, which is in stark contrast to the results of table I that are obtained from traditional priority service pricing theory. This is a powerful aspect of integrating menu design with unit commitment. We further test our model *out of sample* by running it against 1000 scenarios that are drawn from the same distribution, but do not correspond to the scenarios used in the bilevel model. We find that the realized reliability levels amount to 5.9%, 93.4%, 97.8%, 99.7% and 100.0%, and are therefore very close to the *in-sample* reliability levels.

TABLE III
ECONOMIC PERFORMANCE OF PRIORITY SERVICE PRICING (PSP), FLAT TARIFFS (FT) AND REAL-TIME PRICING (RTP).

Policy	Social Welfare (M €)	Consumer Benefits (M €)	Consumer Net Benefits (M €)	Producer Profits (M €)	Producer Costs (M €)
FT	5700.5	6876.8	5234.5	466.1	1176.2
PSP	5763.4	6952.0	5297.4	466.1	1188.6
RTP	5782.3	6992.4	5515.1	267.2	1210.2

Table III compares the economic performance of priority service pricing, flat tariffs and real-time pricing. It is observed that priority service pricing increases social welfare by 1.1% compared to the welfare achieved by a flat tariff. This corresponds to 77.1% of the welfare gains that can be achieved from moving from a flat tariff to real-time pricing. The producer profit under the flat tariff is set as the target profit of the bi-level model. We can achieve this profit target exactly when solving the bi-level model.

4) Interruption Patterns: A powerful feature of our proposed model is that it reveals the interruption patterns associated with a given level of reliability. These interruption patterns are the *direct* output of our model, in particular they correspond to the $d_{i,t,\omega}$ variables. This illuminates the impact of different options on the discomfort that is experienced by households. In Fig. 5 we present the interruption pattern for 1 kW of supply into the four most reliable options of table II.

To illustrate the usefulness of this model, note that although option 2 corresponds to a reliability level of 92.9% (which, if evenly distributed, implies an interruption frequency of 4.4 minutes per hour), around hour 1000 of the simulation, loads under this option experience a *continuous* interruption of 20 hours. The severity of such continuous interruptions needs to be carefully accounted for by utilities, and cannot be furnished by the classical theory of priority service pricing [17]. We note that the excessive stress that the system experiences around hours 500, 1000 and 7000 is driven by three factors: high industrial and commercial demand, low renewable production and low availability of conventional generators due to maintenance.

V. CONCLUSIONS AND FUTURE WORK

We reformulate priority service pricing as a bi-level optimization problem. Our model couples the problem of menu design with unit commitment. We are thus able to extend the classical theory of priority service pricing [17] by including non-convex costs and constraints in the production simulation model. We further develop a decomposition algorithm for solving the problem heuristically, and illustrate its effectiveness on

[11]We consider this gap as being acceptable, based on typical optimality gaps that are used in the stochastic unit commitment literature [33], [34], [42] that are in the order of 1%.

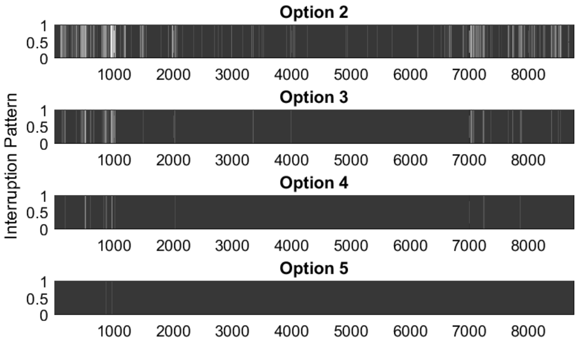

Fig. 5. Interruption pattern of 1 kW power in the most reliable four options for the case study of section IV-B.

a realistic case study of the Belgian electricity market. Our study of the Belgian power system demonstrates that priority service pricing can reap 77.1% of the gains of real-time pricing by using a menu with 5 options.

There are various directions that can be considered in future research. We have outlined various improvements to detailed aspects of the model, such as the endogenous determination of breakpoints and relaxing the requirement that all consumer types choose an option. Moreover, the framework that we develop here can be used for revisiting a host of other nonlinear pricing applications that can be found in the literature, but are currently limited to closed-form solutions that do not exploit the power of commercial solvers [22]–[24], [28]. We are specifically interested in an extension of priority service pricing to multilevel demand subscription, which is a price menu with both capacity and energy charges [28], and a comparison of its performance to the menu presented in this paper. Another direction of interest is to apply the bi-level approach in order to incorporate capacity expansion planning considerations in the design of the menu [22], so as to guarantee that the promised reliability can be delivered on a daily or weekly basis. Finally, we are interested in using the interruption patterns of the proposed model as input to an energy router problem [43] that assesses the impact of priority service pricing on specific devices within a home.

VI. APPENDIX - A TOY EXAMPLE TO ILLUSTRATE ADMM

Consider a system with $\mathcal{L}$ consumers, whose valuation is indicated by $V_l, l \in \mathcal{L}$, and whose power demand is uniformly equal to D. We design a menu with a single option, assuming that the total target subscription is $D = K \cdot (VB_1 - VB_0)$ with valuation breakpoints VB_0 and VB_1. Suppose that the system consists of one generator, which is operational with a probability of P_1 and is out of service with a probability P_2. The generator is assumed to have a capacity of $P_{\max}$ and a marginal cost of MC. The profit target is $\Pi_\star$. The overall model can be formulated as follows:

$$\min_{d_1,d_2,c_1,c_2,r,\pi} \quad P_1 \cdot c_1 + P_2 \cdot c_2 - 0.5 \cdot (VB_0 + VB_1) \cdot (P_1 \cdot d_1 + P_2 \cdot d_2) \quad (69)$$

$$\text{subject to: } 0 \leq d_1 \leq P_{\max} \quad (70)$$
$$d_2 = 0 \quad (71)$$
$$d_1 \leq D \quad (72)$$
$$c_1 = MC \cdot d_1 \quad (73)$$
$$c_2 = 0 \quad (74)$$
$$r \cdot D - (P_1 \cdot d_1 + P_2 \cdot d_2) = 0 \quad (75)$$
$$\pi \cdot D - (P_1 \cdot c_1 + P_2 \cdot c_2) = \Pi_\star \quad (76)$$
$$r \cdot V_l - \pi \geq 0, l \in \mathcal{L} \quad (77)$$
$$0 \leq r \leq 1 \quad (78)$$
$$0 \leq \pi \leq \Pi^+ \quad (79)$$

where d_1 denotes the supply in scenario ω_1 and d_2 in scenario ω_2. The objective (69) minimizes the negative of social welfare. Constraints (70) and (71) require that supply be limited by the available capacity in each scenario. Constraint (72) requires that the supply be limited by the subscription quantity. Constraints (75) and (76) are the reliability and profit constraints, respectively. Constraint (77) guarantees that consumers obtain a non-negative surplus.

We drop d_2 and c_2 since they are equal to 0 and create a copy of d_1 and c_1 (we denote the variable and its copy as d_x and d_z, c_x and $c_z$respectively), yielding

$$\min_{d_x,d_z,c_x,c_z,r,\pi} \quad P_1 \cdot c_x - P_1 \cdot 0.5 \cdot (VB_0 + VB_1) \cdot d_x \quad (80)$$

$$\text{subject to: } 0 \leq d_x \leq P_{\max} \quad (81)$$
$$d_x \leq D \quad (82)$$
$$c_x = MC \cdot d_x \quad (83)$$
$$r \cdot D - P_1 \cdot d_z = 0 \quad (84)$$
$$\pi \cdot D - P_1 \cdot c_z = \Pi_\star \quad (85)$$
$$r \cdot V_l - \pi \geq 0, l \in \mathcal{L} \quad (86)$$
$$0 \leq r \leq 1 \quad (87)$$
$$0 \leq \pi \leq \Pi^+ \quad (88)$$
$$d_x - d_z = 0 \quad (89)$$
$$c_x - c_z = 0 \quad (90)$$

The correspondence between the variables of the model and the stylized formulation in section III is as follows: $\mathbf{x}_1 = (d_x, c_x)$, $\mathbf{x}_2 = (r, \pi)$ and $\mathbf{z} = (d_z, c_z)$. The function $f(\mathbf{x}_1)$ refers to objective (80). The set $\mathcal{C}_1$ corresponds to constraints (81) - (83) while the set $\mathcal{C}_2$ corresponds to constraints (84) - (88). The equalities $A\mathbf{x}_1 - \mathbf{z} = 0$ correspond to constraints (89) and (90). The augmented Lagrangian using scaled dual variables is written as:

$$L_\rho(d_x, d_z, c_x, c_z, r, \pi, u_1, u_2)$$
$$= P_1 \cdot c_x - P_1 \cdot 0.5 \cdot (VB_0 + VB_1) \cdot d_x \quad (91)$$
$$+ \rho/2\big((d_x - d_z + u_1)^2 + (c_x - c_z + u_2)^2\big) \quad (92)$$

The x-update of the update (56) corresponds to solving:

$$(PX): \min_{d_x,c_x} \; P_1 \cdot c_x - P_1 \cdot 0.5 \cdot (VB_0 + VB_1) \cdot d_x \quad (93)$$
$$+ \rho/2\big((d_x - d_z^k + u_1^k)^2 + (c_x - c_z^k + u_2^k)^2\big) \quad (94)$$
$$\text{subject to: } (81) - (83) \quad (95)$$

The z-update which corresponds to (57) is:

$$(PZ): \min_{d_z, c_z, r, \pi} \rho/2\big((d_x^{k+1} - d_z + u_1^k)^2 + (c_x^{k+1} - c_z + u_2^k)^2\big) \quad (96)$$

$$\text{subject to: } (84) - (88) \quad (97)$$

The u-update which corresponds to (58) is:

$$u_1^{k+1} := u_1^k + d_x^{k+1} - d_z^{k+1} \quad (98)$$

$$u_2^{k+1} := u_2^k + c_x^{k+1} - c_z^{k+1} \quad (99)$$

A more detailed description of the toy example with assumed values for parameters is available from [31].

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